COSC 5P05 - Introduction to Lambda-Calculus

Term Test 3

Question 1 (10 marks): Show that $[\![\text{fst}((\lambda x\!:\!A.\langle x, x\rangle)\ M)]\!]_\Delta = [\![M]\!]_\Delta$.

Solution:

$$
\begin{aligned}
&[\![\text{fst}((\lambda x\!:\!A.\langle x, x\rangle)\ M)]\!]_\Delta \\
&= p_1 \circ [\![(\lambda x\!:\!A.\langle x, x\rangle)\ M]\!]_\Delta \\
&= p_1 \circ \text{eval} \circ \langle [\![\lambda x : A.\langle x, x\rangle]\!]_\Delta, [\![M]\!]_\Delta\rangle \\
&= p_1 \circ \text{eval} \circ \langle \Lambda([\![\langle x, x\rangle]\!]_{\Delta\cup\{x\}}), [\![M]\!]_\Delta\rangle \\
&= p_1 \circ \text{eval} \circ \langle \Lambda(\langle p_x, p_x\rangle), [\![M]\!]_\Delta\rangle \\
&= p_1 \circ \text{eval} \circ (\Lambda(\langle p_x, p_x\rangle) \times \text{id}) \circ \langle \text{id}, [\![M]\!]_\Delta\rangle \\
&= p_1 \circ \langle p_x, p_x\rangle \circ \langle \text{id}, [\![M]\!]_\Delta\rangle \\
&= p_x \circ \langle \text{id}, [\![M]\!]_\Delta\rangle \\
&= [\![M]\!]_\Delta.
\end{aligned}
$$

Question 2 (10 marks): Show that $[\![\langle \text{fst}(M), \text{snd}(M)\rangle)]\!]_\Delta = [\![M]\!]_\Delta$ for every λ-term M.
Hint: Recall that $\langle f \circ h, g \circ h\rangle = \langle f, g\rangle \circ h$ and that $\langle p_1, p_2\rangle = \text{id}$.

Solution:

$$\begin{aligned}
[\![\langle \mathrm{fst}(M), \mathrm{snd}(M)\rangle]\!]_\Delta &= \langle [\![\mathrm{fst}(M)]\!]_\Delta, [\![\mathrm{snd}(M)]\!]_\Delta\rangle \\
&= \langle p_1 \circ [\![M]\!]_\Delta, p_2 \circ [\![M]\!]_\Delta\rangle \\
&= \langle p_1, p_2\rangle \circ [\![M]\!]_\Delta \\
&= [\![M]\!]_\Delta.
\end{aligned}$$