COSC/MATH 4P61 - Theory of Computation

Term Test 2

Question 1: (8 marks) Use the Pumping Lemma to show that the language

$$L = \{0^i 1 0^i \mid i \geq 1\}$$

is not regular.

Solution: Assume L is regular and let n be the constant from the Pumping Lemma. Pick the word $w = 0^n 1 0^n$. Using the Pumping Lemma we can write w as $w = xyz$ with $|xy| \leq n$. Therefore $y \neq \epsilon$ consists of only 0's that appear before the 1 in w. The word $xy^0z = xz$ must be in L, which is a contradiction since it contains n copies of 0 after the 1 but less than n copies of 0 before the 1.

Question 2: (12 marks, 3 marks for each part) Consider the following grammar:

$$\begin{aligned} S &\to aA \mid abS \\ A &\to BB \mid CAC \\ B &\to bB \mid \epsilon \\ C &\to aC \end{aligned}$$

a. Eliminate all ϵ-productions.

b. Eliminate all unit productions from the resulting grammar in a).

c. Eliminate all useless symbols from the resulting grammar in b).

d. Put the resulting grammar in c) in Chomsky Normal Form.

Solution:

a. B is obviously nullable. A is also nullable because of $A \rightarrow BB$. We obtain:

$$\begin{aligned} S &\rightarrow aA \mid a \mid abS \\ A &\rightarrow BB \mid B \mid CAC \mid CC \\ B &\rightarrow bB \mid b \\ C &\rightarrow aC \end{aligned}$$

b. There is only one unit production $A \rightarrow B$ so that we obtain:

$$\begin{aligned} S &\rightarrow aA \mid a \mid abS \\ A &\rightarrow BB \mid bB \mid b \mid CAC \mid CC \\ B &\rightarrow bB \mid b \\ C &\rightarrow aC \end{aligned}$$

c. S, A, B are obviously generating; C is not. We obtain:

$$\begin{aligned} S &\rightarrow aA \mid a \mid abS \\ A &\rightarrow BB \mid bB \mid b \\ B &\rightarrow bB \mid b \end{aligned}$$

In this grammar all symbols are reachable, and, hence, useful.

d. We obtain the following grammar in Chomsky Normal Form:

$$\begin{aligned} S &\rightarrow CA \mid a \mid CE \\ A &\rightarrow BB \mid DB \mid b \\ B &\rightarrow DB \mid b \\ C &\rightarrow a \\ D &\rightarrow b \\ E &\rightarrow DS \end{aligned}$$