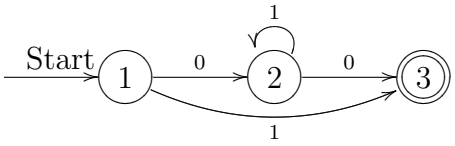
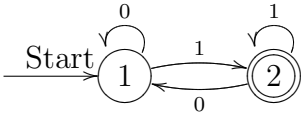
COSC/MATH 4P61 - Theory of Computation

Example Questions Test 1

Question 1: Find a regular expression R so that $L(R) = L(D)$ where D is the following DFA:

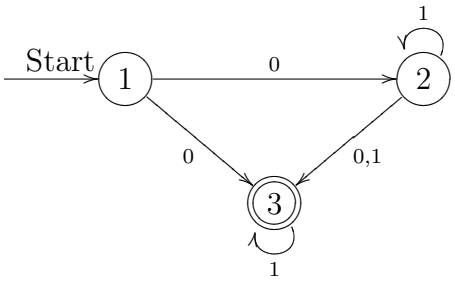


Question 2: Find a regular expression R so that $L(R) = L(D)$ where D is the following DFA:

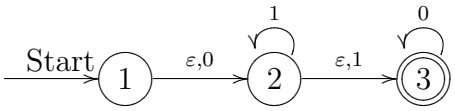


Hint: In computing the expression use the equations (1) $(\varepsilon + X)^* = X^*$ *and (2)* $(\varepsilon + X)X^* = X^*(\varepsilon + X) = X^*$ *and (3)* $Y + X^*Y = X^*Y$ *and (4)* $X + XY^* = XY^*$.

Question 3: Construct a DFA A so that $L(A) = L(N)$ where N is the following NFA:



Question 4: Consider the following NFA N_1 with ε-transitions:



a. Construct a NFA N_2 (without ε-transitions) so that $L(N_1) = L(N_2)$.

b. Construct a DFA A so that $L(A) = L(N_1) = L(N_2)$.

Solutions

Question 1:

$$\begin{aligned}
R &= R_{13}^{(3)} \\
&= R_{13}^{(2)} + R_{13}^{(2)}(R_{33}^{(2)})^* R_{33}^{(2)} \\
&= R_{13}^{(2)} + R_{13}^{(2)}(\emptyset)^*\emptyset \\
&= R_{13}^{(2)} \\
&= R_{13}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\
&= 1 + 01^*0.
\end{aligned}$$

Question 2:
First we have

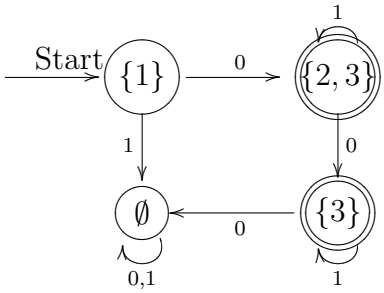
$$\begin{aligned}
R_{12}^{(1)} &= R_{12}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\
&= 1 + (\varepsilon + 0)(\varepsilon + 0)^* 1 \\
&= 1 + (\varepsilon + 0)0^* 1 && \text{by (1)} \\
&= 1 + 0^* 1 && \text{by (2)} \\
&= 0^* 1, && \text{by (3)} \\
R_{22}^{(1)} &= R_{22}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\
&= \varepsilon + 1 + 0(\varepsilon + 0)^* 1 \\
&= \varepsilon + 1 + 00^* 1. && \text{by (1)}
\end{aligned}$$

Then we compute

$$\begin{aligned}
R &= R_{12}^{(2)} \\
&= R_{12}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\
&= 0^*1 + 0^*1(\varepsilon + 1 + 00^*1)^*(\varepsilon + 1 + 00^*1) \\
&= 0^*1 + 0^*1(1 + 00^*1)^*(\varepsilon + 1 + 00^*1) && \text{by (1)} \\
&= 0^*1 + 0^*1(1 + 00^*1)^* && \text{by (2)} \\
&= 0^*1(1 + 00^*1)^*. && \text{by (4)}
\end{aligned}$$

Question 3:

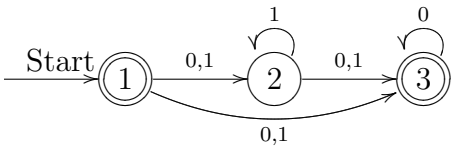
	0	1
$\{1\}$	$\{2,3\}$	$\emptyset$
$\{2,3\}$	$\{3\}$	$\{2,3\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\{3\}$	$\emptyset$	$\{3\}$



Question 4:

a. The new set of final state includes the starting state 1 because there is a path labeled only by ε to the final state 3.

	0	1
1	$\{2,3\}$	$\{2,3\}$
2	$\{3\}$	$\{2,3\}$
3	$\{3\}$	$\emptyset$



b.

	0	1
$\{1\}$	$\{2,3\}$	$\{2,3\}$
$\{2,3\}$	$\{3\}$	$\{2,3\}$
$\{3\}$	$\{3\}$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$