COSC 5P01 – Coding Theory
Winter 2020
Assignment #1

Due Date: Thursday, 13th February, 9:30am (at the start of class)
This assignment accounts for 10% of your final grade and is worth a total of 100 marks.

The purpose of this assignment is to ensure that you understand the basic concepts of coding theory studied so far. To obtain full marks it is necessary to produce complete and precise answers and also to show your work.

Question 1 (10 marks) This question is taken from Problem 1.17 in [1].
Show that in every linear binary self-orthogonal code, either all codewords have weight divisible by 4, or exactly half have weight divisible by 4 and the remainder have weight divisible by 2 but not divisible by 4.

Question 2 (20 marks) This question is taken from Problem 2.11 in [3].

Let C be a linear binary code with parity check matrix

H = 110 000 000
101 000 000
000 110 000
000 101 000
000 000 110
000 000 101

a. (4 marks) What are the parameters *n*, *k* and *d* of C?

b. (4 marks) Give a generator matrix of C.

c. (4 marks) Find the largest integer *t* such that *every* pattern of up to *t* errors will be decoded correctly by a maximum-likelihood decoder for C.

d. (4 marks) A codeword of C is transmitted and the word *y* = 101010101 is received. What will be the output of the decoder when applied to *y*?

e. (4 marks) Given that the answer to part (d) is the correct codeword, how many errors does the decoder correct in this case? How is this number consistent with the value *t* in part (c)?

Question 3 (20 marks) This question is taken from Problems 1.23 and 1.24 in [2].

a. (5 marks) Find eight binary vectors of length 6 so that $d(u,v) \geq 3$ between any pair of them.
Hint: produce a generator matrix for a linear binary (6,3) code and verify that the minimum weight (= minimum distance) is satisfied for all codewords.

b. (15 marks) Show that it is not possible to find 9 binary vectors of length 6 so that $d(u,v) \geq 3$ between any pair of them. Note that such a code would not be linear and you *cannot* assume it would build upon the code produced in part (a). You may choose to answer this question by mathematical arguments, by computer search, or by some combination of the two. If you use a computer search, it must be well explained and exhaustive (since you wish to show non-existence).

Question 4 (25 marks)

Consider the following generator matrix for a linear binary (12, 6, 4) code.

```
G =   100000 111000
      010000 110100
      001000 110010
      000100 110001
      000010 101111
      000001 011111
```

a. (4 marks) Give a parity check matrix for this code.

b. (15 marks) Give a syndrome table for this code. All coset leaders should be listed in lexicographical order, together with their corresponding syndromes. To save space, syndromes can be given as row vectors rather than column vectors. You can use a computer program to produce this table if you wish; if you do, then you must hand in the source code.

c. (6 marks) For each of the following received vectors *y*, give the resulting codeword *x* obtained by applying syndrome decoding. Again, if a computer program is used, you must hand in the source code.

- *y* = 111111 111111
- *y* = 000011 100000
- *y* = 001100 110000

Question 5 (15 marks) This question is taken from Exercise 80 in [1].

Consider the problem of constructing a ternary (3-ary) (4,2,3) perfect code.

a. (3 marks) Provide a generator matrix for such a code.

b. (3 marks) Give its weight enumerator.

c. (9 marks) For every codeword, give the list of words in its sphere.

Question 6 (10 marks) This question is taken from Exercise 80 in [1].

The weight of a coset is defined as the weight of its coset leader.
Prove that a perfect binary 1-error-correcting linear code of length n has exactly 1 coset of weight 0 and n cosets of weight 1, and that there are no other cosets. Hints: how many vectors are there of weight 1 in $GF(2)^n$? Could distinct vectors of weight 1 be in the same coset?

Submission Requirements:
All of the following must be handed to the instructor directly:
1. A cover sheet, available from `http://www.cosc.brocku.ca/forms/cover`, completely filled out. Your assignment will not be marked unless one is submitted with the assignment.
2. Full and complete, typed answers to all questions.
3. Commented and properly documented printouts for all programs (if used).
4. You must also submit your assignment electronically. To do this, create a directory on Sandcastle containing all files for this assignment, and run the script `submit5p01` from this directory.

References:

1. W.C. Huffman and V.Pless, Fundamentals of Error Correcting Codes, Cambridge University Press, 2003.
2. V.Pless, Introduction to the Theory of Error-Correcting Codes, 3rd ed., Wiley-Interscience Series in Discrete Mathematics and Optimization, 1998.
3. R.M.Roth, Introduction to Coding Theory, Cambridge University Press, 2006.