Schema Refinement and Normal Forms

Chapter 19

The Evils of Redundancy

- *Redundancy* is at the root of several problems associated with relational schemas:
  - ***redundant storage:*** some information stored repeatedly.
  - ***update anomalies:*** inconsistency if one copy is updated but others aren't.
  - ***insertion anomalies:*** certain information can't be stored unless other unrelated information stored too.
  - ***deletion anomalies:*** certain information can't be deleted without losing other unrelated information too.

Avoiding Redundancy

- Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.
- Main refinement technique: *decomposition* (replacing ABCD with, say, AB and BCD, or ACD and ABD).
- Decomposition should be used judiciously:
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

Functional Dependencies (FDs)

- A functional dependency $X \rightarrow Y$ holds over relation R if, for every allowable instance *r* of R:
  - $t1 \in r,\ t2 \in r,\ \pi_X(t1) = \pi_X(t2)$ implies $\pi_Y(t1) = \pi_Y(t2)$
  - i.e., given two tuples in *r*, if the X values agree, then the Y values must also agree. (X and Y are *sets* of attributes.)
- An FD is a statement about *all* allowable instances.
  - Must be identified based on semantics of application.
  - Given some allowable instance *r1* of R, we can check if it violates some FD *f*, but we cannot tell if *f* holds over R!
- K is a candidate key for R means that $K \rightarrow R$
  - However, $K \rightarrow R$ does not require K to be *minimal* so it might be a *superkey*.

Example: Constraints on Entity Set

- Consider relation obtained from Hourly_Emps:
  - Hourly_Emps (*ssn*, *name*, *lot*, *rating*, *hrly_wages*, *hrs_worked*)
- *Notation*: We will denote this relation schema by listing the attributes: SNLRWH
  - This is really the *set* of attributes {S,N,L,R,W,H}.
  - Sometimes, we will refer to all attributes of a relation by using the relation name. (e.g., Hourly_Emps for SNLRWH)
- Some FDs on Hourly_Emps:
  - *ssn* is the key: $S \rightarrow SNLRWH$
  - *rating* determines *hrly_wages*: $R \rightarrow W$

Example (Contd.)

Problems due to R ⟶ W :

- *Update anomaly*: Can we change W in just the 1st tuple of SNLRWH?
- *Insertion anomaly*: What if we want to insert an employee and don't know the hourly wage for his rating?
- *Deletion anomaly*: If we delete all employees with rating 5, we lose the information about the wage for rating 5!

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

S	N	L	R	H
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Hourly_Emps2

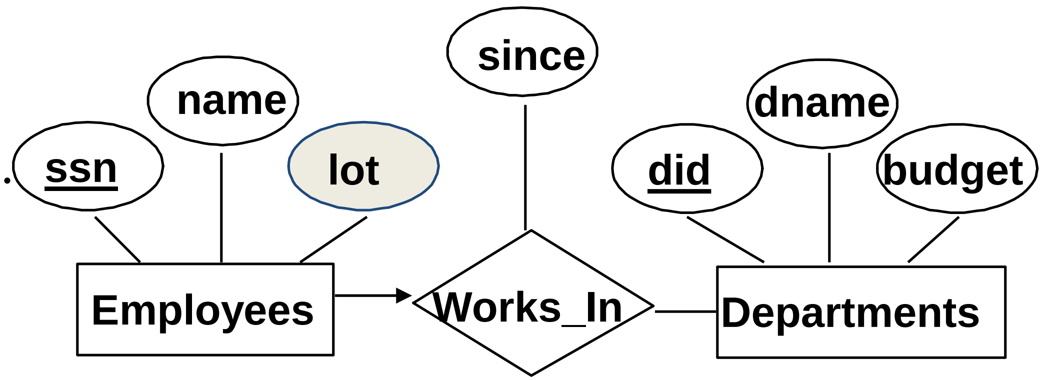
R	W
8	10
5	7

Wages

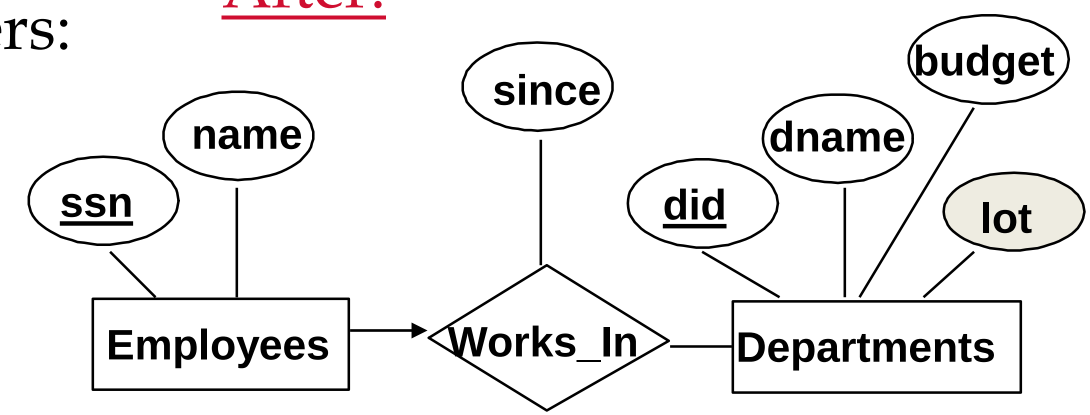
Refining an ER Diagram

- 1st diagram translated into:
  Workers(S,N,L,D,I)
  Departments(D,M,B)
  - Lots associated with workers.
- Suppose all workers in a dept are assigned the same lot: D → L
- Redundancy; fixed by decomposition of Workers:
  Workers2(S,N,D,I)
  Dept_Lots(D,L)
- Can fine-tune this:
  Workers2(S,N,D,I)
  Departments(D,M,B,L)

Before:



After:



Reasoning About FDs

- Given some FDs, we can usually infer additional FDs:
  - $\{ssn \rightarrow did,\ did \rightarrow lot\}$ implies $ssn \rightarrow lot$
- An FD f is *implied by* a set of FDs F if f holds whenever all FDs in F hold.
  - F^+ = *closure of F* is the set of all FDs that are implied by F.
- Armstrong's Axioms can be applied repeatedly to find all FDs implied by F.

Armstrong's Axioms (X, Y, Z are sets of attributes):

- *Reflexivity*: If $Y \subseteq X$, then $X \rightarrow Y$
- *Augmentation*: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
- *Transitivity*: If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$
- These are *sound* rules: they generate *only* FDs in F+
- These are *complete* rules: they generate *all* FDs in F^+

Reasoning About FDs (Contd.)

- Additional rules that follow from Armstrong's Axioms:
  - *Union:* If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Decomposition:* If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
- Example: Contracts(*cid,sid,jid,did,pid,qty,value*), and:
  - C is the key: $C \rightarrow CSJDPQV$
  - Project purchases each part using single contract: $JP \rightarrow C$
  - Dept purchases at most one part from a supplier: $SD \rightarrow P$
- $JP \rightarrow C$, $C \rightarrow CSJDPQV$ imply $JP \rightarrow CSJDPQV$
- $SD \rightarrow P$ implies $SDJ \rightarrow JP$
- $SDJ \rightarrow JP$, $JP \rightarrow CSJDPQV$ imply $SDJ \rightarrow CSJDPQV$

Reasoning About FDs (Contd.)

- Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in number of attributes!)
- Typically, we just want to check if a given FD $X \rightarrow Y$ is in the closure of a set of FDs F.
- An efficient check:
  - Compute *attribute closure* of X (denoted X^+) wrt F:
    - Set of all attributes A such that X $\rightarrow$ A is in F^+
    - There is a linear time algorithm to compute this.
  - Check if Y is in X^+

Attribute Closure X^+

$X^+ = X$;

repeat until no change {

for each FD $U \rightarrow V$

if U is in X^+

then add V to X^+;

}

- Does $F = \{A \rightarrow B,\ B \rightarrow C,\ C\,D \rightarrow E\}$ imply $A \rightarrow E$?
  - i.e, is $A \rightarrow E$ in the closure F^+ ?
  - Equivalently, is E in A^+ ?

Finding Keys using FDs

- If X is a key for R, then:
  - $X \rightarrow R$ (or equivalently, $X^+ = R$), and
  - this is not true for any proper subset of X
- "Algorithm" to find all keys:

for each single attribute X {
    if $X^+ = R$ then X is a key
}
repeat for all sets of 2 attributes not containing a key, sets of 3 attributes… until no more possibilities exist.

- Shortcuts:
  - attributes *never* on the RHS of any FD *must* be in key
  - attributes *only* on the RHS of FDs *cannot* be in key

Normal Forms

- Returning to the issue of schema refinement, the first question to ask is whether any refinement is needed!
- If a relation is in a certain *normal form* (BCNF, 3NF etc.), it is known that certain kinds of problems are avoided/minimized. This can be used to help us decide whether decomposing the relation will help.
- Role of FDs in detecting redundancy:
  - Consider a relation R with 3 attributes, ABC.
    - No FDs hold: No possible redundancy.
    - Given $A \rightarrow B$: Several tuples could have the same A value, and if so, they'll all have the same B value!

Boyce-Codd Normal Form (BCNF)

- Reln R with FDs *F* is in BCNF if for all X→ A in F^+
  - A ∈ X (this is called a *trivial* FD), or
  - X contains a key for R.
- In other words, R is in BCNF if the only non-trivial FDs that hold over R are key constraints.
  - No redundancy in R can be detected using FDs alone.
  - If we are shown two tuples that agree upon the X value, we can infer the A value in one tuple from the A value in the other.
  - If example relation is in BCNF, the 2 tuples must be identical (since X is a key).

X	Y	A
x	y1	a
x	y2	?

Third Normal Form (3NF)

- Reln R with FDs *F* is in 3NF if for all X $\rightarrow$A in F^+
  - A $\in$ X (called a *trivial* FD), or
  - X contains a key for R, or
  - A is part of some key for R.
- *Minimality* of a key is crucial in third condition above!
- If R is in BCNF, obviously it is also in 3NF.
- If R is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no ``good'' decomp, or performance considerations).
  - *Lossless-join, dependency-preserving decomposition of R into a collection of 3NF relations always possible.*

What Does 3NF Achieve?

- If 3NF is violated by X $\rightarrow$A, one of the following holds:
  - X is a proper subset of some key K ("partial dependency")
    - We store (X, A) pairs redundantly.
  - X is not a proper subset of any key.
    - There is a chain of FDs K $\rightarrow$ X $\rightarrow$ A ("transitive dependency"): we can't associate X value with a K value unless we also associate an A value with an X value.
- But: even if reln is in 3NF, these problems could arise.
  - e.g., Reserves SBDC, S $\rightarrow$ C, C $\rightarrow$ S is in 3NF, but for each reservation of sailor S, the same (S, C) pair is stored.
- Thus, 3NF is indeed a compromise relative to BCNF.

Decomposition of a Relation Scheme

- Suppose that relation R contains attributes A_1 ... A_n. A *decomposition* of R consists of replacing R by two or more relations such that:
  - Each new relation scheme contains a subset of the attributes of R (and no attributes that do not appear in R), and
  - Every attribute of R appears as an attribute of one of the new relations.
- Intuitively, decomposing R means we will store instances of the relation schemes produced by the decomposition, instead of instances of R.
- E.g., Can decompose SNLRWH into SNLRH and RW.

Example Decomposition

- Decompositions should be used only when needed.
  - SNLRWH has FDs S $\rightarrow$ SNLRWH and R $\rightarrow$ W
  - Second FD causes violation of 3NF; W values repeatedly associated with R values. Easiest way to fix this is to create a relation RW to store these associations, and to remove W from the main schema:
    - i.e., we decompose SNLRWH into SNLRH and RW
- The information to be stored consists of SNLRWH tuples. If we just store the projections of these tuples onto SNLRH and RW, are there any potential problems that we should be aware of?

Problems with Decompositions

- There are three potential problems to consider:
  1. Some queries become more expensive.
     - e.g., How much did sailor Joe earn? (salary = W*H)
  2. Given instances of the decomposed relations, we may not be able to reconstruct the corresponding instance of the original relation! (Decomposition is not "lossless-join")
     - Fortunately, not in the SNLRWH example.
  3. Checking some dependencies may require joining the instances of the decomposed relations. (Decomposition is not "dependency-preserving")
     - Fortunately, not in the SNLRWH example.
- *Tradeoff*: Must consider these issues vs. redundancy.

Lossless Join Decompositions

- Decomposition of R into X and Y is *lossless-join* w.r.t. a set of FDs F if, for every instance *r* that satisfies F:
  - $\pi_X(r) \bowtie \pi_Y(r) = r$
- It is always true that $r \subseteq \pi_X(r) \bowtie \pi_Y(r)$
  - In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- Definition extended to decomposition into 3 or more relations in a straightforward way.
- *It is essential that all decompositions used to deal with redundancy be lossless! (Avoids Problem (2).)*

More on Lossless Join

- The decomposition of R into X and Y is lossless-join wrt F if and only if the closure of F contains:
  - $X \cap Y \rightarrow X$, or
  - $X \cap Y \rightarrow Y$
- In particular, the decomposition of R into UV and R – V is lossless-join if $U \rightarrow V$ holds over R.

A	B	C
1	2	3
4	5	6
7	2	8

A	B
1	2
4	5
7	2

B	C
2	3
5	6
2	8

A	B	C
1	2	3
4	5	6
7	2	8
1	2	8
7	2	3

Dependency Preserving Decomposition

- Consider CSJDPQV, C is a key, JP $\rightarrow$ C and SD $\rightarrow$ P.
  - BCNF decomposition: CSJDQV and SDP
  - Problem: Checking JP $\rightarrow$ C requires a join!
- Dependency preserving decomposition (Intuitive):
  - If R is decomposed into X, Y and Z, and we enforce the FDs that hold on X, on Y and on Z, then all FDs that were given to hold on R must also hold. *(Avoids Problem (3).)*
- *Projection of set of FDs F*: Suppose R, with set of FDs F, is decomposed into X and some other relation(s). The projection of F onto X (denoted F_X) is the set of FDs U $\rightarrow$ V in F^+ (*closure of F*) such that U, V are in X.

Dependency Preserving Decomposition Cont.

- Decomposition of R into X and Y is ***dependency preserving*** if $(F_X \text{ union } F_Y)^+ = F^+$
  - i.e., if we consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+.
- Suppose R with set of FDs F is decomposed into
  $R_1, \ldots, R_n$ with sets of FDs $F_1, \ldots, F_n$
- Algorithm to check if decomposition is dependency-preserving:

```
preserving = true;
for each FD X⟶Y in F {
    if Y is not in X+ over F1 union F2 union … Fn
        then preserving = false;
}
```

Dependency Preserving Decomposition Cont.

- It is important to consider F^+, not F, in the definition of projection of a set of FDs F:
  - ABC, $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$, decomposed into AB and BC.
  - Is this dependency preserving? Is $C \rightarrow A$ preserved?????
- Dependency preserving does not imply lossless join:
  - ABC, $A \rightarrow B$, decomposed into AB and BC.
- And vice-versa! (Example?)

Decomposition into BCNF

- Consider relation R with FDs F. If X $\rightarrow$ Y violates BCNF, decompose R into R - Y and XY.
  - Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
  - e.g., CSJDPQV, key C, JP $\rightarrow$ C, SD $\rightarrow$ P, J $\rightarrow$ S
  - To deal with SD $\rightarrow$ P, decompose into SDP, CSJDQV.
  - To deal with J $\rightarrow$ S, decompose CSJDQV into JS and CJDQV
- In general, several dependencies may cause violation of BCNF. The order in which we ``deal with'' them could lead to very different sets of relations!

BCNF and Dependency Preservation

- In general, there may not be a dependency preserving decomposition into BCNF.
  - e.g., CSZ, CS $\rightarrow$ Z, Z $\rightarrow$ C
  - Can't decompose while preserving 1st FD; not in BCNF.
- Similarly, decomposition of CSJDQV into SDP, JS and CJDQV is not dependency preserving (w.r.t. the FDs JP $\rightarrow$ C, SD $\rightarrow$ P and J $\rightarrow$ S).
  - However, it is a lossless join decomposition.
  - In this case, adding JPC to the collection of relations gives us a dependency preserving decomposition.
    - JPC tuples stored only for checking FD! (*Redundancy!*)

Decomposition into 3NF

- Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
- To ensure dependency preservation, one idea:
  - If $X \rightarrow Y$ is not preserved, add relation XY.
  - Problem is that XY may violate 3NF! e.g., consider the addition of CJP to `preserve' $JP \rightarrow C$. What if we also have $J \rightarrow C$?
- Refinement: Instead of the given set of FDs F, use a *minimal cover* for F (also called a *canonical cover*).

Minimal Cover for a Set of FDs

- *Minimal cover* G for a set of FDs F:
  - Closure of F = closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- Intuitively, every FD in G is needed, and ``*as small as possible*'' in order to get the same closure as F.
- Note: *minimal cover* is also sometimes called *canonical cover*.

Minimal Cover, cont.

- Steps to compute minimal cover:
  1 decompose FDs so that each has only one attribute on RHS
  2 minimize LHS of each FD
  3 delete redundant FDs:
    - an FD $X \longrightarrow Y$ is redundant if X^+ under F- $\{X \longrightarrow Y\}$ contains Y
- Example:

  $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$ has the following minimal cover:
  - $A \rightarrow B$, $ACD \rightarrow E$, $EF \rightarrow G$ and $EF \rightarrow H$

Decomposition into 3NF, cont.

- Using the minimal cover, we obtain a Lossless-Join, Dependency-Preserving Decomposition
- Options:
  – same algorithm as for BCNF, except using minimal cover and checking for 3NF conditions, or
  – 3NF synthesis:
    - for each FD $X \longrightarrow A$ in the minimal cover, create a relation XA
    - if no relation contains a key, then add a relation containing just the attributes in a key
    - refinement: relations corresponding to FDs with the same LHS can (and should, for efficiency) be combined

Summary of Schema Refinement

- If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
  - Must consider whether all FDs are preserved. If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
  - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.