COSC 4P98 Lecture notes: **Math, Noise, Fractals, and Music**
November 13, 2017
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- Automated composition: computer algorithms create music (see "Algorithmic Composition" topic).
- Using mathematics to generate musical melodies has a long history. Composers used dice to help create new melodies.
- Can be used as a source of interesting composition ideas
  - Often more successful as a help tool, rather than a source of entire composition!

1. Random algorithm 1: dice (white noise)

- Use a random number generator to generate successive notes.
- Note that there are different distributions of random numbers. Ones used in programming languages try to follow a uniform distribution. Each random number is equally likely in a sequence (but of course, it is actually a formula/code creating it, so it is predictable).
- Example code:

```
For i =1 to 1000
        note(i) = random(1,88)
        duration(i) = random(1, 4)
```

- Result: bad! Random gibberish. Too chaotic, no patterns.
- This is equivalent to using "white noise" to select notes. White noise is uniform randomness.
  - Probability of selecting a particular note (frequency) is the same as all the others. There is no correlation between different frequencies selected.
  - spectral density: $Pr(f) = 1 / f^0 = 1$ (for all frequencies f)
    - where f is frequency difference between successive notes

2. Random algorithm 2: random walk (brown noise)

- Code:

```
Start at a note.
Loop until done:
        Flip a coin.
        If heads --> move up a note.
        else move down a note
```

- Alternatively: could use a "roulette wheel" spinner, marked with 0, +1, +2, +3, -1, -2, -3
- Result: Boring! Too correlated, not enough novelty. Meandering around.
- This is called "brown noise". It has lower frequencies (rates of change) than white noise.
- spectral density: $pr(f) = 1 / f^2$
  - meaning: proportion of change f is $1 / f^2$

- higher changes (bigger f) are less likely!

3. Random algorithm 3: Fractals (pink noise)

- This lies between white noise and brown noise
- spectral density: $pr(f) = 1 / f^1 \quad = \quad 1 / f$
  - again, f is change between successive frequencies
- Result: most musical. Some patterns, some chaos.
- Algorithms to generate pink noise are more complex.
  - An approximate algorithm:
    - let there be 3 bits
    - Also 3 dice, called R, G, B: sums range from 3 to 18 (16 tones)
    - list binary numbers: 000, 001, ..., 111
    - Assign a die to each column (e.g. R, G, B)
    - Start at 000: roll 3 dice, and start with that tone
    - Next number is 001, so 1 bit (Blue) changed. Roll that die only, leave other dice. THe new sum is next tone.
    - Next number is 010, so G and B change: Roll those dice (leave R).
    - Continues this way.
  - Result is more random than Brown noise, but less random than White noise.
- 1/f noise is a famous distribution.
  - fractal in nature: long-term correlation of signals.
  - magnify a fractal: see same structures. "Scaling".
  - Seen throughout nature:
    - frequencies of floods
    - coast line lengths
    - electronic circuit behaviour
    - traffic
    - speech
- Most important to us, it appears when real musical intervals are studied.
  - Clark and Voss (1978) studied real music intervals. They fall into a 1/f distribution.

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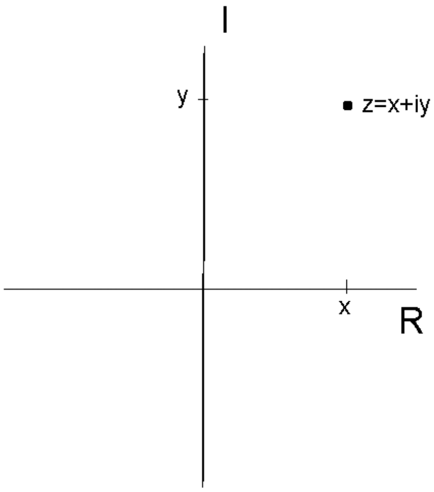
4. Fractals and Music

- Recall imaginary numbers:



$$i = \sqrt{-1}, \quad i^2 = -1, \quad |z| = \sqrt{x^2 + y^2}$$

- Mandelbrot set: $z_{n+1} = z_n^2 + \lambda$
  where $z_0 = 0 + 0i$ and λ=complex number.
    - n is an index or counter of a loop
    - Idea is that some points stay at their location, others escape.
    - if a number iin the loop reaches 2, it escapes: goes to infinity
    - keep track of |z|, and use max limit for n
    - if point hasn't reached 2 by max, then assume it won't
        - else record the n in which it reaches 2.
    - Plot: use n as an index into a colour map
    - lambda is the (x,y) coord of plot being tested.
        - --> familiar plot of Mandelbrot fractal.
- Other kinds of equations are studied too (eg. Julia sets)
- Music
    - Mandlebrot plots have 1/f distributions of frequencies
    - can therefore use to generate note change information (like pink noise earlier)
    - Main issue: mapping the graphical characteristics to musical events
        - Fractal image: (x,y) -> (R,G,B)
        - Music: pitch, note change, timbre, note duration, amplitude, ...
    - lots of research into fractal music.
    -

References:

White, Brown, and Fractal Music. Martin Gardner, *Fractal Music, Hypercards, and more.* Freeman, 1992.